



Australian Academy of Science

DESKTOP REVIEW OF
**MATHEMATICS SCHOOL EDUCATION
PEDAGOGICAL APPROACHES AND
LEARNING RESOURCES**

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Desktop Review of Mathematics School Education Pedagogical Approaches and Learning Resources

Submitted by the Australian Academy of Science

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FOREWORD

The Australia Government Department of Education and Training commissioned the Australian Academy of Science to undertake a desktop review of the evidence relating to gaps in current pedagogical approaches and learning resources for the teaching of mathematics to inform the Mathematics by Inquiry initiative. As such, the Academy was asked to restrict its focus to a set of research questions rather than covering all aspects of pedagogy and resources. The Department commends this paper for its insight into the gaps in pedagogical approaches and learning resources in mathematics in Australia.

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EXECUTIVE SUMMARY

In order to meet the mathematical demands of the 21st century, there is a need for national attention to broaden Australia's focus from numeracy to achieving the full set of mathematical proficiencies outlined in the Australian Curriculum Mathematics (ACM), especially problem solving and reasoning, productive disposition and the accompanying AC general capabilities. We must aim to develop mathematical capabilities that are perceived by learners as powerful and genuinely useful in the present and future, through learning experiences students generally find engaging and that offer opportunities for exploration, explanation and creativity. This has been a long-term goal of mathematics teaching, but it must now be achieved for most of the school population. There are no quick fixes. Attention is required to the many layers of the education system.

The review finds that achieving the full set of proficiencies requires the adoption of the spirit of inquiry-based pedagogy (Q.2), although for mathematics teaching inquiry is conducted within 'pedagogically well-engineered' problem situations and lesson formats. Well documented pedagogies (Q.1) are available to deepen learning, build capacity to solve problems creatively and develop affective, behavioural and cognitive engagement in students (including those in underperforming groups (Q.5)). Lessons will frequently use mathematically-able software and special digital learning objects (Q.1). Assessment methods need to be revised to include problem solving and reasoning prominently and with status (Q.3). Countries which are performing at the top of international assessments show how problem solving and reasoning can be built into lessons (Q.4). In multiple ways, they also show how co-ordinated, long term and well-researched programs working with curriculum, resources and teachers can achieve better mathematics learning.

The report finds that the teaching of mathematics is quite well supported by ACM-linked resources, but through a combination of the resources themselves, inappropriate pedagogy, and the way in which resources are used in schools, the depth of the Australian Curriculum is missed (Q6–13). This especially relates to problem solving and reasoning proficiencies, including the ability to use mathematics to solve real world problems and to tackle problems of some complexity. Addressing this lies behind all of the recommendations. Additionally report identifies three specific gaps in resources. Beyond this, there are overarching difficulties experienced by all teachers in assembling coherent programs from disparate sources of variable quality, and most keenly felt by the many teachers with insufficient professional knowledge for teaching mathematics.

Broad recommendation

Institute a long-term Australian initiative of co-ordinated resource development and evaluation, research and professional learning for system-wide improvement in mathematics education. Within this program, mathematical inquiry, problem solving and reasoning are promoted in multiple ways.

Specific recommendations

Three major projects and 8 smaller projects are recommended in priority order, within this co-ordinated initiative. Three further recommendations relate to the ACM. The full list of recommendations is at the end of this report.

- Major Project 1 on solving real world problems mathematically is designed to fill a gap related to PISA’s survey of mathematical literacy, with lessons learned from high-achieving countries.
- Major Project 2 uses assessment and associated resources (consolidated, refreshed or created) to drive a richer treatment of the ACM problem solving and reasoning proficiencies.
- Major Project 3 draws together resources to build teachers’ mathematical pedagogical content knowledge, which has been found to be the major knowledge-related determinant of teacher effectiveness. Out-of-field teachers are one of the target groups.
- Projects 4 and 7 are reviews of current programs for disadvantaged and indigenous students, so that practical, educationally sound advice can be given to schools to assist in decision making.
- Projects 5 and 8 are to fill gaps identified for secondary mathematics, both capitalising on new technologies. (Encouraging reasoning and proof in geometry; using mathematically-able software).
- Project 6 provides support for creating excellent whole school programs from the overwhelming multitude of resources available. Useful protocols for rating resources will be investigated.
- Project 9 aims to make available to all teachers the excellent work of state projects for early numeracy.
- Project 10 provides a review of the growing body of research evidence on achieving large scale change for mathematics.
- Project 11 is to create higher education award courses for out-of-field teachers that focus on a deep understanding of the content that they need to teach and how students learn those ideas.
- The three recommendations for the ACM are for:
 - regular systematic review,
 - improved description of the problem solving and reasoning proficiencies, and
 - setting up a task force to design an alternative senior mathematics course that focuses on 21st century mathematical applications.

INTRODUCTION

In the 21st century knowledge economy, the mathematical demands for a prosperous society are greater than ever before. This means that people at all levels of the workforce are likely to need more mathematical¹ skills than before. The Stockholm Declaration: Mathematics for the 21st century (Center for Curriculum Redesign, 2013) called for “far deeper and reconceptualized understanding of mathematics by the entire population as a critical right”; the strengthening of equity so that everyone is given the best chance to use mathematics to real advantage in their personal, occupational and societal roles; and a systematic and long term rethink of the branches and topics that are included in the mathematics curriculum.

In line with this call, this document argues that it is time to broaden the focus of national attention from numeracy (which is often conceived quite narrowly) towards achieving the full set of proficiencies set out in the Australian Curriculum: Mathematics (ACM) richly interpreted. Additionally the seven AC general capabilities (especially ICT capability and Critical and creative thinking) anticipate that mathematics will contribute to the development of children’s ability to be thoughtful, strategic and active in and beyond school. This has long been a goal of mathematics education in Australia and internationally, but has only ever been patchily achieved. With the proceeding Australia-wide adoption of the ACM, a national effort moving beyond numeracy is timely. This will involve changing the widespread perception that doing mathematics well is *just* being able to follow rules to get answers to arbitrary short exercises quickly and accurately. Such perceptions are currently held by many students and parents and some teachers, and are evident in some popular mathematics resources and assessments. Instead, we must aim to develop mathematical capabilities that are perceived by learners as powerful and genuinely useful in the present and future, through learning experiences that are found by many students to be engaging, offering opportunities for exploration and creativity. This is a long-term goal without quick fixes, and attention is required across the many layers of the education system.

Mastering mathematics involves inter-twined but different types of proficiency. Following the ACM, these are: (i) *conceptual understanding* referring to a reasoned knowledge of the objects and processes of mathematics, and the complex webs of connections between them (ii) *procedural fluency*, the ability to calculate, do algebra, draw graphs etc. accurately and quickly (iii) *problem solving* ranging from solving simple word problems through to having strategic skills to conduct non-routine mathematical investigations and being able to develop mathematical models to solve real world problems and (iv) *reasoning*, so that students appreciate that mathematical rules are not arbitrary but are logically derived, and so that they can engage in the deductive thinking that characterises mathematics. In addition, learning mathematics well, and being willing and expecting to use it effectively beyond school, requires the development of a *productive disposition* (Kilpatrick, Swafford & Findell, 2001, p.116). All four proficiencies and productive disposition need attention and there is evidence that Australian achievement within each could be improved.

In the process of this review we have identified a strong consensus that problem solving and reasoning proficiencies are the highest priorities, with high priority specific needs also identified for conceptual understanding and for curriculum review. There is great energy

¹As in the ACM, this document uses the word ‘mathematics’ to include statistics.

and enthusiasm put into the improvement of mathematics teaching in schools and an enormous number of resources are available although some are hard to find. This, combined with the variable quality (especially of online resources) makes it difficult, especially for beginning teachers and out-of-field teachers, to choose high-quality lesson resources and assemble them cohesively. We have also found that very few programs have been evaluated in a robust way.

Our over-arching recommendation is for a well-funded, well-evaluated long-term coherent approach. Germany (see Q.4) demonstrates that this can achieve results in a federal system. The coherence is (a) to consolidate disparate resources, (ii) to have a consistent approach to pedagogy, (iii) to avoid duplication and capitalise on what exists and (iv) to have a long term practical agenda.

PEDAGOGY

Question 1: Are the current pedagogical approaches to teaching mathematics relevant and/or appropriate in the 21st century knowledge economy, including the extent to which they reflect the contemporary technology rich environment?

The range of current pedagogical approaches that are available for mathematics teaching that have been promoted, researched and put into practice around Australia are appropriate for the development of the mathematical literacy required for the 21st century. However, all experts agree that the majority of classrooms in both primary and secondary schools reduce such pedagogies to focus on a narrower range of skills than are proposed by the ACM and narrower than is needed for the future. In terms of the four ACM proficiencies there is insufficient emphasis on problem solving and reasoning. Australian teachers exhibit a great concern for the welfare of the students in their classes and aim to match their teaching to the diverse backgrounds and needs they see, but sometimes the implemented priorities match the past rather than the future. Fortunately, there is some excellent practice in both primary schools and secondary schools to build on.

The USA National Council of Teachers of Mathematics (NCTM, 2014) is concerned that students achieve deep learning, because the mathematical demands of the 21st century go well beyond routine calculations and in any case, these can now be done with computers. They report strong research that “student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural in nature” (p.17). The PISA 2012 results indicate that classroom pedagogy can influence students’ drive and willingness to engage with complex problems: “Teachers’ use of cognitive-activation strategies, such as giving students problems that require them to think for an extended time, presenting problems for which there is no immediately obvious way of arriving at a solution, and helping students to learn from the mistakes they have made, is associated with students’ perseverance and openness to problem solving” (OECD, 2013, p. 187). Willingness to engage with complex problems and perseverance are aspects of productive disposition. There are many other reports that show achieving the mathematical outcomes citizens now require, needs pedagogies that at least substantially deepen the current common practices, and sometimes largely refocus it.

Motivating students to engage in mathematics is now recognised as a key issue for pedagogy. Stephens (2011) notes that ‘in those classrooms where the focus is perceived to be on rewarding [only] those who are successful at mathematics, it is easy for those who see themselves as unsuccessful to opt out. Likewise, those who see mathematics as difficult or confusing, or as a source of failure and criticism, are likely to lose interest’ (p.1). (See also Q.5). Stephens cautions that apparent ‘engagement’ may be only ‘busy work’—students working on their own, perhaps with individual assistance from the teacher, to complete repetitious routine exercises from worksheets or textbooks. Stephens emphasises that having students working together on mathematical tasks is essential to reveal connections between mathematical concepts and make mathematical thinking explicit, so the ‘busy work’ approach to engagement comes at a cost. This is consistent with the findings of Rollard (2012) and Sullivan (2015) discussed in Q.7 that show

pedagogies which focus on challenging problems (at an appropriate level) and inquiry can deepen student engagement.

There are also strong similarities here with the work of Tytler et al. (2008, p. 34) that presents the key pedagogical principles from the Middle Years Pedagogy Research and Development project (MYPRAD). The first principle is that “students are challenged to develop deeper levels of understanding; emphasising student questioning and exploration, and engagement with significant ideas and practices”. Other principles describe productive learning environments (promoting engagement) and teaching strategies that are fortunately a good match for the focus on reasoning and problem solving recommended above. Tytler’s principles apply broadly across the levels of schooling.

Moving towards more widespread adoption of such pedagogies requires long term and substantial teacher professional development. Most recent research literature on students’ mathematics achievement asserts that teacher quality (hence lesson quality) is the most important school factor influencing student learning. Mathematics teachers need mathematics content knowledge (MCK, the knowledge of mathematics that they share with adults in other mathematically demanding professions), pedagogical knowledge (PK, the knowledge of students and teaching practices that they share with teachers of other subjects) and a special blend of mathematics and teaching knowledge called mathematics pedagogical content knowledge (MPCK) that includes knowledge concerning student mathematical thinking and development, curriculum goals and sequencing, resources, ways of explaining, good representations etc. As is shown under Question 3, MPCK is perhaps the major key to success, but it is inadequately developed in many teachers. Strengthening MPCK is the highest priority for the associated teacher development. (See Project 3).

It is important to note that the four proficiencies of the ACM summarise different aspects of rounded mathematical achievement. Different lesson structures are required to teach these different proficiencies, and part of MPCK is to be able to operationalise these differences. A good school mathematics program includes these different types of learning coherently and in a balanced way. We know of no research evidence that would show how many schools have such a program. The TIMSS teacher surveys, for example, do not ask directly relevant questions. The need to assist schools to create balanced programs is recognised by some states. For example, Issue 5 (March 2015) of the newsletter *Mathematical Bridge* from the NSW Department of Education and Communities to schools addresses essential features of a mathematics program for Stages 3 and 4.

New digital technologies

The mathematics pedagogies required for the 21st century must include digital resources more prominently than now, to include new methods for mathematical computation, to support content teaching and to increase student engagement. There are three very different implementations of digital technology in the mathematics classroom. First it can be a communication device (e.g. digital textbooks, teacher presentations, student-student collaboration, using the internet for information about definitions etc.). This use is shared with other subjects and is not reviewed here, except to recommend the potential for easy access to real data for problem solving.

Second, there is mathematically-able software on hand-held calculators and computers, (e.g. spreadsheets, dynamic geometry, computer algebra). Their existence has changed how mathematics is done and subsequently, the curriculum content (although the ACM will need regular revision in this regard). Bakker, Hoyles, Kent and Noss (2006) refer to

‘techno-mathematical literacies’ to describe mathematics as it exists in modern workplaces. For most adults, doing mathematics and working with the workplace computer systems are indivisible. Workers do not need to be able to do challenging computation (although mental computation remains important) but they need a deeper understanding of the mathematical structures behind the data that is being reported (for example) to carry out their work intelligently. In schools, this means that students need to use mathematically-able software (on calculators, computers or tablets) to assist with routine procedures of mathematical work so they can focus on formulating problems mathematically and understanding how the mathematical model behaves. For example, primary school students can use a calculator to assist in more complex problem solving and software to visualise data. By Year 12, near professional digital tools can be used (e.g. advanced graphics calculators with statistics & algebra). Digital technology has changed the priorities and content of mathematics, and on-going work with teachers is required to assist them to adjust curriculum goals what solution techniques they teach.

Third, rich resources of special software, designed for teaching mathematics at every level, deserve to be more widely used in teaching and to be built coherently into school programs. They can make practice fun with immediate feedback, help teachers demonstrate and explain and students visualise, and meet expectations of today’s digital natives. Many products are single purpose ‘applets’ (for computer or tablet) which assist in teaching a particular concept. For example, they may demonstrate the balance model for solving equations, and set students appropriate problems to solve with the balance and/or with algebra, sometimes in a game format. See Question 6 for sample websites. There are appropriate resources available for all four proficiencies that should be used to enhance all aspects of mathematics learning, increasingly as schools get better technology access.

TIMSS 2011 (Thomson, Hillman & Wernert 2012) reported that computer use in Australia is high internationally, with about two thirds of students of Year 8 students having regular access to, and 78% using, computers in mathematics lessons. We can expect these numbers to have increased in the years since the 2011 survey. About half of Year 8 students reported using computers to practise skills at least once per month and also about a half for explorations. However, the nature and purpose of computer use is critical. Question 12 discusses some of what is needed to improve use.

Recommendation. All the projects within the co-ordinated initiative would promote the pedagogies and uses of digital technologies shown to be successful above. Project 3 is especially focused on developing MPCK.

Question 2: What is the role of inquiry-based pedagogy in the teaching of mathematics (including across the age groups and across mathematical disciplines)?

If students are to achieve all of the goals of the ACM, then an appropriate version of inquiry-based pedagogy is essential, especially for the problem solving proficiency, but also for the ‘21st century skills’ represented in the ACM and reinforced in the AC general capabilities—communication of mathematical ideas and products (including with

technology), effective collaboration in working mathematically, orientation to ask questions, frequently experiencing the opportunity for creativity in doing mathematics (e.g. finding multiple ways to solve problems) and sharpening critical thinking. We want students to be creative problem solvers and critical thinkers, so we absolutely must give broad and regular experience of doing this and help students learn from the experience.

However, the term ‘inquiry-based pedagogy’ is problematic for mathematics education and terms such as a problem solving approach or an investigative approach are more commonly used. The pedagogy of inquiry-based learning is founded on the principle that students should be actively and socially engaged in the process of learning, constructing new concepts based on their current knowledge and understanding. Inquiry-based learning, as described in the research literature, often refers to highly student driven approaches where the student decides the questions to ask, the research methods to use, and different learning occurs for different students. This very open student-led interpretation of inquiry-based pedagogy has only a very small place in mathematics.

Instead the best investigative pedagogies for mathematics use ‘well engineered’ mathematical problems, where engagement in the problem solving process individually and with others and supported by the teacher will assist in the development of targeted concepts, or strategic skills, or the ability to transfer knowledge. There is a spectrum of purposes and thus variations in the pedagogy are required. A few outcomes within the ACM can be well served with quite open inquiry-based learning (e.g. experience in statistical investigation). More topics will use a structured investigation spreading over say two lessons so that students explore a particular concept, or undertake a real world mathematical modelling task, to experience how to identify the relationships involved and make appropriate assumptions to apply their mathematical knowledge in real situations. Some investigations will be designed to focus on new content learning and others will focus on developing strategic skills; with other variations due to student age and interests. In this sense inquiry builds strongly on the four proficiency strands of the ACM and it can be tied quite rigorously to the three PISA processes of *Formulating*, *Employing* and *Interpreting* (OECD 2010). There are many research studies which demonstrate the benefits of an investigative approach, for developing strategic and problem solving skills, for encouraging deeper conceptual learning and for improving engagement (e.g., Boaler 1998).

Science by Doing has adopted a modified form of inquiry learning, so that its general resources (e.g. on questioning) are certainly relevant for mathematics. However investigations in mathematics need to be more highly structured to be successful. In science, memorable rubrics to guide teachers thinking about lesson structure have been successful—these are needed for mathematics too.

There is much excellent Australian material to support an investigative approach (e.g. maths300 <http://www.maths300.esa.edu.au/>) and appropriate Australian research (e.g. Sullivan et al. 2015), especially at lesson level. However, we need to look overseas for models of comprehensive curricula. The very successful and well researched *Connected Mathematics Project* (CMP) (<https://connectedmath.msu.edu/>), for example, was funded by the U.S.A.’s National Science Foundation. Designed around inquiry-based learning, CMP covers content and process goals (problem solving, reasoning and proof, communication, connections and representations) for Years 6–8. Lowe (2004) noted that early teachers often supplemented CMP materials with more skills practice, underlining the need for a comprehensive mathematics curriculum to attend appropriately to different

types of learning. A purely investigative approach is not advocated for mathematics. The underlying principle of reflective inquiry applying across lessons is described by Hiebert et al. (1996) as “allowing students to wonder why things are, to inquire, to search for solutions, and to resolve incongruities. It means that both curriculum and instruction should begin with problems, dilemmas, and questions for students.” (p. 14). In recommending an inquiry-based pedagogy in this document, we use ‘inquiry’ in this sense of investigation, which will usually be very carefully designed for its purpose.

Although there are obvious benefits of inquiry-based learning, the issue of teachers’ mathematical pedagogical content knowledge is critical. Teachers must know how to scaffold students’ learning without giving away answers, and how to intervene appropriately when students’ use incorrect mathematical reasoning. Stein et al. (1996) have shown in detail how such events often derail higher-order thinking in classrooms. These are special skills must be the focus of professional development. The Mathematics Assessment Project (MAP) and Bowland Mathematics (see below) are models of how this professional development for teachers can be linked into student resource.

Inquiry-based learning: Recent overseas projects

The following projects also provide models of resources for investigative learning for Australia (theoretical and practical basis for lesson design, combining student and teacher learning, and harnessing assessment for change as is recommended in Project 2).

Realistic Mathematics Education (RME) (www.fi.uu.nl/en/rme/) is a successful problem-solving based program, used in all schools in The Netherlands. It has an excellently researched theoretical background and a deep but teacher-friendly philosophy that could well be used for in Australian resource development. It is referred to as ‘realistic’ in that children learn through engaging in solving problems in meaningful contexts. Lessons begin with a well-chosen real situation within which is embedded the target mathematics, which is gradually revealed. The stages of progressive horizontal and vertical mathematisation give Dutch teachers a common structure for teaching every topic. RME has influenced education around the world, e.g., the USA and Indonesia.

The *Mathematics Assessment Project* (MAP) (www.map.mathshell.org/) is led by Malcolm Swan. It has developed formative assessment lessons and summative tests with associated professional development materials, for USA students aged 12–18. The materials include a large bank of lessons for formative assessment (some focused on conceptual understanding, others on non-routine problem solving); professional development modules to help teachers with the new pedagogical challenges of fully meeting the specifications of the USA’s Common Core, and sample summative tests designed to help teachers and students monitor their progress. Australian projects could also achieve teacher learning through student materials, and drive change by harnessing assessment

The Bowland Mathematics project (<http://www.bowlandmaths.org.uk/>) from the same group as MAP uses multimedia to present real world problems for mathematical modelling over several lessons. It also includes professional development based on the student resources.

Cornerstone Maths (www.cornerstonemaths.co.uk/) is one example of a technology-based innovation with an inquiry focus, adapted for the UK from US work that originated many years ago with James Kaput (Hoyle, Noss, Vahey and Roschelle, 2013). Special software features motion of characters on the screen that is dynamically linked to graphs, and to

tables and equations to help students more easily visualise how changes to any one affect the others. There is exciting potential in special software like this and more will become available. We know that some Australian schools used early versions of Kaput's software within normal teaching but we know of no school which made the comprehensive rethink of Years 7 – 10 algebraic functions teaching that the software supports. Attractive though it is, there are no specific recommendations about developments like this, because we think it is best to start with research rather than full scale development projects.

The EU's Promoting Inquiry-based learning in Mathematics and Science (PRIMAS) (<http://www.primas-project.eu>) project uses inquiry-based learning for both science and mathematics, but as above, finds that the phrase should be interpreted rather differently for the two subjects (so that the mathematics resources are problem-solving investigations that extend prior teaching of concepts). Extensive professional development accompanied the implementation.

Schoenfeld and Kilpatrick (2013), writing in the context of the PRIMAS project, caution about unrealistic expectations, noting that “efforts to raise the numbers of pupils studying and continuing in mathematics [a stated PRIMAS goal] have not typically been made by changing the nature of instruction in school mathematics” (p. 901). They also discuss scaling up innovations, the need for extensive professional learning and differences between an inquiry approach for maths and science.

Recommendation. All the projects within the co-ordinated initiative would promote an appropriate form of inquiry learning. Existing Australian resources can be consolidated and refreshed in several of the projects. For example, Project 2 draws on the MAP in multiple ways.

Question 3: Do current approaches to assessment in mathematics need to be revised to better reflect problem-solving pedagogical approaches?

The answer to this question is yes. The Australian Association of Teachers of Mathematics (AAMT, 2008), along with many others, notes that “each assessment task is effectively a contract between the teacher and the student that communicates and reflects what is valued in the mathematics classroom”. Similarly, Black et al. (2012) note that if they are not assessed, formal expectations “remain ‘paper expectations’ and will receive little attention in the classroom”. For this reason, it is essential that assessment reflects the full goals of mathematics education. However, all the evidence indicates that NAPLAN now dominates the Australian assessment scene, to Year 9 (Carter 2015). The origins of NAPLAN and the practicalities of testing every student in Australia economically of time and money means that its strength is in assessing procedural fluency with some conceptual understanding. This substantially narrows student experience by implicitly sending the message to schools that more substantial problem solving and reasoning tasks are not important.

The assessment tasks that formed part of the Victorian Certificate of Education Mathematics assessment in the 1990s illustrate significant mathematical problem-solving tasks and investigations that should be an important part of school and teacher assessment. These tasks, undertaken over multiple weeks, clearly endorsed a broader ‘band-width’ of

mathematical performance than could be expected in a traditional timed test, allowing students to try out approaches and write a final report, so communication could be assessed. The assessment criteria, as well as being linked to the award of grades, anticipated the kind of mathematical performances that were to be cultivated by teachers and hence needed to be reflected in the curriculum. Students themselves needed to clearly understand the criteria and their interrelationships in order to structure their reports. The ‘ripple effect’ study by Barnes, Clarke and Stephens (2000) described the profound impact of these Year 11 and 12 assessment criteria and practices in shaping the design of problem-solving tasks, teaching strategies, assessment and reporting in Years 7 – 10. Later when the VCE assessment dropped these components, they also disappeared from Years 7 – 10.

All of this is evidence that when assessment values a more comprehensive ‘band-width’ of mathematical performance, schools will strive to achieve it. Now in Australia, NAPLAN addresses a narrow band-width, and schools respond. The Mathematics Assessment Project is a current example of harnessing assessment change (the USA Common Core) to promote higher-order goals. To date, there have been over 3 million downloads of the 100 lessons (personal communication). The Netherlands’ RME philosophy, which ensures students learn through and are assessed on solving real world problems, presumably contributes to its consistently high PISA performance.

In upper primary and junior secondary years, we should also expect students and teachers to make more use of self- and peer-assessment in developing a culture of reflective practice and in fostering confidence and competence in problem solving, moving students into roles normally used by teachers, the intended outcome of which is to raise levels of learning (Black et al. 2012).

Recommendation. Project 2—Assessment to drive a more complete interpretation of ACM.

Question 4: What can we learn from overseas about the teaching of mathematics that is relevant to education in Australia, particularly from those countries which are performing better than Australia in TIMSS and PISA?

Whilst Australian students have consistently performed above the OECD average for mathematical literacy in the PISA surveys, there has been a steady and significant decline in Australia’s score on the fixed PISA scale from 524 (2003), 520 (2006), 514 (2009) to 504 in 2012. The TIMSS Year 8 assessment (Thomson, Hillman & Wernert 2012) remains unchanged from 1995 to 2011, showing a long ‘tail’, relatively few students achieving at the highest levels, and about one third reporting they are not engaged in their lessons. The gap between the lowest and highest achieving students in Australia was greater than the OECD average, as was the proportion of students reporting classrooms with students not listening, noise and disorder, and teachers needing to wait a long time for students to quieten (Thomson, De Bortoli & Buckley 2013).

PISA 2012 defined three mathematical processes to analyse mathematical literacy: *Formulating* situations mathematically, *Employing* mathematical concepts, facts,

procedures and reasoning, and *Interpreting*, applying and evaluating mathematical outcomes. These reflect the stages of mathematical modelling—using mathematics to solve real world problems. Australian students performed best on *Interpreting* and poorly (relative to Australian overall performance) on *Formulating* and *Employing* subscales. Whilst improvement in all three subscales is desirable, we have particular concern with the ability of Australian students to formulate situations mathematically. This is the first stage in applying mathematics to real world situations and the stage which cannot be taken over by mathematically-able digital technology. Fortunately practices in high-performing countries especially give us clues about how we can improve in *Formulating*. Unlike in Australia, many Asian high-achieving countries emphasise developing students' capacity for mathematical formulation throughout school, with methods that we could also use.

China and Japan

One notable point of contrast between Australian classrooms and those in certain high-achieving countries is that teachers in those countries spend considerably more time working through and exploring one or two rich problems in a lesson rather than on 'busy work' on many simple problems. This latter tendency is clearly endorsed in many Australian lessons which focus on skills applied only in routine situations, and frequently done without genuine cognitive engagement.

In China and Japan, by contrast, teacher and students often spend much of a mathematics lesson on one carefully chosen problem related to concepts previously learned. These problems are not routine applications. They are very frequently situated in real life contexts and require students to work in groups and as a whole class to formulate the problem in suitable mathematical terms. There will usually be a range of appropriate formulations. Students are expected to identify the mathematical aspects of the problem and to represent these using appropriate variables, symbols, diagrams and models. In classroom discussions, students are expected to connect the context-specific language of a problem to the symbolic and formal language needed to represent it mathematically and through the lessons move towards more advanced strategies.

Germany: Generating improvement in PISA performance

A model for action for Australia comes from Germany's response to its poor results in the 1995 TIMSS and later in PISA. Germany had traditionally prided itself on what it believed was a strong and quality education system, but international comparisons revealed otherwise. It was quickly decided that classroom mathematics generally focussed only on learning facts and procedures that were important for the next written test. A very substantial long-term program of co-ordinated research and professional development was initiated with universities, state and federal education authorities, schools and thousands of teachers. There is good evidence that it has resulted in steady improvement. Some highlights of the program are described by Prenzel, Blum, & Klieme (2015). PISA scores rose from 503 in 2003 and 504 in 2006 to 513 (2009) and 514 (2012). As an example of further positive changes in the performance of German students, the proportion of variation in mathematics performance that could be attributed to differences in socio-economic status decreased from 24% in 2003 to 17% in 2012.

An early large-scale part of the program aimed at changing the underlying culture of mathematics and the character of mathematics tasks to include a much broader range of mathematical competencies (problem solving, communication, reasoning, representation, etc.). Materials were shared, tested and improved so that teachers could integrate them into

their own lessons. The program was evaluated carefully (including by using mathematics items and questionnaires from PISA) and extended from secondary schools to primary schools and thousands of teachers, influencing teacher training, curriculum development, articles in teacher magazines and textbooks, all of which no doubt contributed to Germany's mathematics improvement (Prenzel et al. 2015).

Another major research study in the program (see Baumert et al. 2010) aimed to understand why German students performed poorly in PISA (and therefore the all-important mathematical literacy) and how outcomes could improve. They identified, then focussed on, aspects of teaching that stimulated students' thinking and cognitive engagement with lessons. Baumert et al. showed that it is MPCK that is the most important predictor of student learning, and that this greater learning is brought about through provision of cognitive activation and individual learning support. Relevant content knowledge (MCK) is essential to the extent that it enables correct instruction to be given and it also enables the development of MPCK. Evidence showed that differences in MPCK and MCK resulting from different types of pre-service education had persisted across the entire teaching career—experience working as a teacher is not by itself enough to make up for gaps.

Finland: equity and prestigious careers

Finland's director of education Pasi Sahlberg, speaking on ABC Lateline (Sahlberg, 2012), stated that “one of the keys of our good performance is that we have systematically focused on equity and equality in our education system, and not so much on excellence and achievement like many other countries have done.” The Finland Ministry of Education and Culture (2014) claims success is based on high-quality teachers and teacher education.

Singapore: long term focus on the fundamental goals of mathematics education

A key lesson from Singapore comes from its long term focus on problem solving as the centre of the curriculum from 1991 to the present day (Kaur 2014). They recognised that fundamental changes take a long time to implement, even in a small country, and have seen steady improvement towards this goal. They begin work on formulating problems in early primary, and have developed visual methods to do this. As with other Asian countries, curriculum is systematically reviewed at regular intervals (6 years in Singapore). We recommend this predictable approach for Australia, to balance the updating and refinement with the need for stability so that long term effort goes into improving student learning rather than organisational issues.

Recommendations. The overarching recommendation of co-ordinated research and development arises especially from the example of Germany. Project 1 is inspired by the more active way in which some countries pay explicit attention in teaching to formulating mathematical models.

Question 5: Which pedagogical approaches have been shown to work with specific groups underrepresented in advanced mathematics at senior secondary level (girls, Indigenous, disadvantaged students)?

It is now well known that there are very substantial differences between the achievement of Indigenous and non-Indigenous students, and between students of high and low SES. There are small differences in average achievement between girls and boys at age 15. The success of the funding of policy and initiatives implemented in the late 20th century to address the lower participation of girls in mathematics (e.g. Holton et al. 1999) is evidence that targeted interventions can improve the achievement of otherwise disadvantaged groups. The recent growing gender gap also highlights that continuous effort is required.

In general, we firmly believe that good pedagogies for teaching mathematics are good for all students. However, certain characteristics of mathematics present special problems for certain classes of students. Below we present these characteristics and pedagogical responses to them with examples. Later we briefly consider each group of students.

1. Mathematics is a hierarchical subject, where new learning builds on earlier learning in a highly connected way. Hence learning gaps created by absence from school or by unstable school staffing seriously affect progress. The *YuMi Deadly* program (www.ydc.qut.edu.au/) reports its success with Indigenous and low SES groups by providing multi-year programs for secondary students with a very low numeracy base to bring them from about Grade 3 level to age appropriate level in time to embark on senior mathematics.

2. Mathematics has a highly connected web of concepts and skills, so these have to be firmly consolidated to provide a basis for new learning. Homework is important for consolidation and developing procedural fluency, so providing a suitable space and/or tutorial assistance after school can make a substantial difference for some students. The Quicksmart program from U.N.E. has a special focus on developing fluent number facts (procedural fluency) in class. It reports improvements measured by ACER tests over a 30 week program. Students cannot effectively master advanced concepts (e.g. ratio) if they have little number sense (so cannot see factors, for example) so their attention on the new learning is continually interrupted by the challenge of the un-mastered subtasks (e.g. simple division). <http://simerr.une.edu.au/quicksmart/pages/index.php>

3. Many students in Australia, including some Indigenous students and many low SES students of NESB background, do not have an adequate command of English for learning mathematics. This needs particular pedagogies which pay attention to special mathematics words and to the grammar that describes mathematical relationships. Concrete materials and visual methods can also supplement verbal demonstrations. There are successful programs using the students' home language to support their learning with the assistance of peers, the teacher or teaching aides. *Talking Namba* (<http://www.talkingnamba.net/>) appears to be an excellent resource which could be used in a range of situations, although designed for remote Indigenous schools. It includes a series of videos of activities in local languages and English to help Indigenous Assistant Teachers to develop a greater understanding of mathematics and of pedagogy.

4. In mathematics, it is generally clear whether you are right or wrong, and this can discourage students who find it hard, and promote a strong competitive atmosphere in classrooms. As a result, many students develop poor self-concept in mathematics, with a

resulting loss of confidence and often effort. More girls than boys fall into this category. Strategies to overcome this include reducing overt competition in classrooms, introducing more collaborative ways of learning and altering assessment, e.g., to focus on “personal best”. The research literature now includes many detailed studies of how this can be done.

5. Mathematics is inherently abstract, and in this sense is ‘about nothing’. Paradoxically, this gives it power in a vast number of realms of human endeavour. However, it falls to the teacher, in their choice of real world illustrations of mathematical ideas, to make sure that students feel that mathematics is somehow connected with their lives and offers them some power. There are materials to assist teachers to do this for girls, including the awareness of the issue in ordinary textbooks. In remote locations, links with the local community are essential. Brown (2008) advocates a “contextualised mathematics curriculum that will engage indigenous students to actively investigate, analyse, and reflect on real world problems they are interested in.” (p. 97). Lack of self confidence in mathematics and alienation from the abstract content leads to disinterest in STEM careers. Many programs to promote STEM careers have had measureable impact, but more remains to be done.

Indigenous students

The lower participation and achievement in mathematics observed amongst indigenous students is mediated by factors such as remoteness, low SES, low educational background of parents biasing the home environment to be not conducive to study, and home language(s) other than English. Consequently, Indigenous students may require assistance in all the areas above, which perhaps explains the very large achievement gap (Dreise & Thomson 2014). Dreise and Thomson also report that “Indigenous young people are more likely (53 per cent) than non-Indigenous students (41 per cent) to identify family demands and other problems impacting on the time they spent on school work” (p. 3) and that they have lower confidence and are more likely to be ‘anxious’ about it. Dreise and Thomson note that where indigenous students have been successful, the schools have a culture that embraces the community, have high expectations of success for both staff and students, strive for continuous improvement, and have learning environments that can respond to individual needs. As part of the AAMT *Make it Count* national project, teachers in the Alberton cluster of schools in South Australia have attempted to develop culturally responsive mathematics pedagogy to eliminate the feelings of alienation mentioned above. Thornton, Statton and Mountzouris (2012) reported that the approach seems to be ‘impacting positively on those aspects of students’ learning behaviour that we have described as mathematical resilience.’ (p. 734). Meaney, McMurchy-Pilkington and Trinick (2012) note that Indigenous Teaching Assistants can link home and school.

Socioeconomic background

The gap in PISA 2012 scores between students in the highest and lowest SES quartiles is around two-and-a-half years of schooling. This large gap tends to be addressed mainly through whole school programs (e.g. homework support, school engagement, role models and career aspiration, improving low self-esteem) rather than mathematics-specific pedagogies. Special programs (such as the YuMi Deadly program above) to make up for accumulated gaps in their knowledge before making choices about senior secondary mathematics could offer some low SES students a valuable second chance. Students of low SES have been shown to perform better and are more likely to complete year 12 if they attend a high SES school (Caldas & Bankston 1997; Lim, Gemici & Karmel 2013). If students go to different schools on the basis of socioeconomic criteria, then the relationship

between SES and achievement will be strengthened. This is likely to be the current situation in Australia.

Gender

The PISA 2012 results indicate that Australia's mathematical literacy performance has declined more for girls than for boys so that "Australia [is now] further away from providing all students with the same educational opportunities.' (Thomson 2013). The PISA 2012 report (OECD 2015) notes that girls tend to underachieve compared to boys when they are asked to formulate situations mathematically and that girls tend to have less self-confidence than boys in their ability to solve mathematics or science problems, and suggests that certain methods of teaching mathematics can help narrow the gender gap in performance. This is a very well researched issue, and the necessary classroom actions are well understood. For example, girls are also generally understood to prefer co-operation to competition and successful classroom practices have been built around these tendencies. There have been successful interventions by deliberately including real world problems with a social flavour (e.g. Jablonka 2003) to increase interest and motivation. There are still issues with continuing stereotyped community views (e.g. Leder, Forgasz & Jackson 2014). Similarly low expectations are also likely to bedevil the education of some low SES and Indigenous students.

Lower girls' enrolments in senior mathematics reflect career interests. Approximately one-third of females in Australia reported in the PISA Student Questionnaire that they did not think mathematics was important for later study compared with one-fifth of males. (Thomson, de Bortoli & Buckley 2013). Girls as a group show less interest in mathematics-related careers so there are special career programs in most states. For example, The University of Queensland *Women in Engineering* program claims success with women now making up almost 25% of the 2014 engineering undergraduate intake.

No further action specifically for girls is recommended because a large 5 year program, funded by BHP Billiton Foundation, is beginning this year at AMSI. <http://amsi.org.au/2015/04/29/bhp-backs-maths-for-girls/> The target audience for *Choose Maths* is primary, secondary and tertiary female students, their parents, teachers and the general public. Choose Maths plans a national program of professional development, career awareness, networks, and awards for teachers to turn around community attitude to participation in mathematics.

Recommendations. Projects 4 and 7 are designed to provide best information to schools about the programs that are available for disadvantaged students to catch up with mathematics and for indigenous students in regional or remote communities. Providing well founded advice on these programs has been beyond the scope of the present review. There is no specific gender recommendation. All projects within the co-ordinated program would attend to equity and engagement in classrooms.

CURRICULUM RESOURCES (QUESTIONS 6-12)

General comments on resources

Systematic searching for resources by people knowledgeable in the field shows that there are very few aspects of the ACM where there are insufficient resources, although there is a perception of gaps. There are three issues. (i) Some teachers lack knowledge of where good resources might be found, and most lack time to find them. These are especially young teachers and out-of-field teachers working in the far-too-numerous schools without a very well-documented whole school mathematics program in place. (ii) Many resources (especially free internet resources) are of poor quality. (iii) It is a major undertaking to choose resources from disparate sources and make a coherent, well sequenced program from them.

One highly experienced, but obviously busy primary teacher emailed on this topic: *“teachers spend a huge amount of time finding resources...there are many really great tasks in various places (often hard to recall where they saw them) but if those tried and true great rich open ended deep tasks were linked to actual aust curric levels somehow in one publication/document it would save a huge amount of search and planning time”*.

Quality assurance for educational resources is tackled in several ways overseas. In some countries, schools must use the official textbook. In others there are systems for approval of textbooks. In the USA, for example, curriculum materials are presented by publishers to be judged for adequacy by committees (experts and sometimes parents) against criteria which reflect the intention of the state standards and sometimes also some technical criteria (e.g. not more than a certain percentage of pages of revision of previous years' work). School districts then formally adopt curriculum resources and teachers are expected to follow them closely. This is very different from the very open practices in Australia, where teachers/schools are expected to draw on multiple resources. With online publishing, the number of these is now becoming enormous. Instead, if we want to provide advice on resources, we need to be highly selective about recommended resources and provide detailed information about their purpose and features.

Recommendations. Project 6 is devised to assist with preparation of whole school programs. A dedicated resource rating project is not recommended, but consideration needs to be given to the nature of information that teachers require, from digital banks such as ESA's Scootle.

Question 6: How well is the teaching of mathematics across primary and secondary school supported by existing resources linked to the Australian Curriculum?

We have found that the teaching of mathematics is quite well supported by ACM-linked resources, but through a combination of the resources themselves and the way in which they are used in schools they do not cover the ACM in depth.

Commercially-produced source/activity books (mainly primary) or textbooks (all of which are increasing digital) provide the most important resource for many teachers and students, especially because of ease of use, internal coherence and an assurance that the mathematical content of the ACM is covered. Some are accompanied by a teacher guide and additional remedial, practice or enrichment material. As well as explanatory text and practice exercises, most textbooks include some real-world applications and problem-solving activities. However, two issues related to the ‘shallow teaching syndrome’ arise. Two secondary school studies, the 1999 TIMSS Video Study (<http://www.timssvideo.com/timss-video-study>) of classrooms and a later examination of textbooks (Vincent & Stacey 2008) have shown that in Australia, there is an internationally high proportion of problems of low procedural complexity being used, with considerable repetition, and an absence of deductive reasoning. Whilst problems of low complexity are good for initial learning, there also must be challenging problems. However, Australian teachers tend to break down potentially challenging problems down into very small steps so that students have very little opportunity for mathematising, formulation and communication. Most textbooks provide explanations for most topics rather than presenting ‘rules without reasons’, but this appears to be only to justify the rule to use in the practice exercises. The explanations are not subsequently used as thinking tools (Stacey & Vincent 2009). Moreover, there is widespread ‘textbook teaching’ where teachers skip reasoned explanations, merely demonstrate procedures, and then supervise exercise practice, hence missing most of the depth of the ACM. This form of teaching is not confined to textbook users, nor to secondary school teachers, and it should not be taken as a criticism of the publications per se. It is just as prevalent using worksheets or teacher-constructed lessons, which may not even have the advantage of a cohesive program. The shallow teaching syndrome needs attention through curriculum materials and in teacher professional development.

A wide range of digital and print resources linked to the Scope and Sequence of the Australian Curriculum is available from Education Services Australia via Scootle. However, these resources have been ‘retro-fitted’ to the Australian Curriculum rather than being designed specifically for it, so resources are not always a good match. At this stage there is incomplete coverage. Some of the resources only incidentally involve the mathematics to which they are tagged, which makes teachers’ searching more difficult. Descriptions could also be made more informative in the future. There is now considerable variation in quality (mathematical and technical). In time, and with strong attention to quality, Scootle should become an excellent resource. It is now open to teacher education students so they will learn to use it from the start. It already incorporates mathematics ‘applets’ from leading sites such as Utah State University’s *National Library of Virtual Manipulatives* (nlvm.usu.edu).

There is still a shortage of good resources in some strands, for example secondary geometry digital resources that focus on geometric reasoning rather than simply demonstrating properties.

For the early years of primary school, the teacher resources developed in the *Count Me In (Too)* project(s) in NSW from about 1995 and variants of this project in other states and in New Zealand (Higgins & Parsons, 2009) provide an excellent foundation for the ACM and could be consolidated and made available to all Australian teachers, although states may prefer to support their own versions. Western Australia has the *First Steps in Mathematics* series (<http://det.wa.edu.au/stepsresources/detcms/navigation/first-steps-mathematics/>)

which provides illustrated frameworks for student progress from early years through primary school. New Zealand has also extended the early years approach through compulsory years (see the very comprehensive nzmaths site nzmaths.co.nz).

The *Top Drawer* resources available from the website of the Australian Association of Teachers of mathematics (AAMT) need expansion (e.g., to better cover problem solving). The Asia Education Foundation (www.asiaeducation.edu.au) has a limited but very good selection of mathematics resources which explicitly link to Asian cultures.

Mathletics (<http://www.mathletics.com.au>) is the most popular commercial website for mathematics in Australia (3.5 million students worldwide), and one of a range of commercial online services now being purchased by schools (and families) with explicit links to the Australian Curriculum. The main strength of the site is in assisting students to master skills and algorithms, especially in number and algebra. Many students like to compete on speed and accuracy with students from other schools. A key feature of commercial resources is that they increasingly offer learning management features for teachers, including automatic marking, report generation, and tracking of students' progress through content levels. The major international textbook companies are also beginning to combine textbook material, digital learning objects and learning management features, in order to make teachers work easier. Similar products will come to Australia. Services such as Scootle cannot do this. Some school authorities provide a little advice on commercial resources. For example, SA's Department of Education and Child Development lists Mathletics as one of the direct instruction products useful for students with gaps in their learning (http://www.ican.sa.edu.au/files/links/Assessment_for_Learning_an.pdf). Tasmania's Department of Education lists it as a product purchased under a 'National Partnerships Low SES Implementation Plan' grant.

The major gap identified is in the teaching of problem solving and reasoning, from Year 3 up, with engaging extended problems that students can tackle, often collaboratively, as a significant part of their mathematics experience. The problems must include both mathematical investigations and applications to the real world (mathematical modelling), for which there are many fine teacher resources (including several written by the authors of this report) which provide the raw material to fill this gap. Many are only available commercially, for example, *maths300*, the hands-on boxed tasks of the *Mathematics Task Centre* <http://mathematicscentre.com/taskcentre/> and the Australian Mathematics Trust enrichment for upper primary and middle years' students. There are overseas websites with many very high quality resources, e.g., the USA's NCTM Illuminations (illuminations.nctm.org), the UK's NRICH <http://nrich.maths.org/frontpage> and the Freudenthal Institute's wisweb (http://www.fi.uu.nl/wisweb/applets/mainframe_en.html) and rekenweb (http://www.fisme.science.uu.nl/publicaties/subsets/rekenweb_en/). Teachers need assistance to maintain a focus on higher-order thinking when using such activities (e.g. Smith & Stein 2011).

Recommendations. Projects 5 and 8 are to fill specific gaps identified above. Projects 1 and 2 address the gaps in the teaching of problem solving and reasoning. Project 3 builds teacher capacity.

Question 7: To what extent do existing resources engage the full range of students (girls, Indigenous, different learning styles)?

Modern Australian resources almost uniformly demonstrate strong awareness of the need to be appealing and inclusive to boys and girls, to students with many different ethnicities, and with different out-of-school interests. In mathematics resources this is evident in features such as the topics of word problems and the people in illustrations. That being acknowledged, all the evidence (e.g., Stephens 2011; PISA and TIMSS surveys) indicates that there are problems with engagement in all three of its inter-connected aspects – *affective* (whether students feel good, like it), *behavioural* (e.g., whether they actually do the work) and *cognitive* (whether they think deeply and persistently about the tasks that are set). Low engagement is prevalent, from the middle years to the end of compulsory mathematics for the groups listed but is more widespread. For example, we have observed that able students are very commonly bored by mathematics lessons, because they (correctly in our opinion) find the work trivial. Able students need more challenging work, which must be addressed through the richer curriculum experience advocated throughout this report.

Lack of engagement is partly (certainly not fully) due to classroom culture and teaching and assessment norms (hence affected by teaching resources). Rollard (2012) reviewed a large number of studies on the middle years of schooling, and found that these years are critical for the formation of student attitudes (e.g. persistence, taking risks) and that good outcomes relate to classroom culture, especially by valuing mastery of the content itself rather than just valuing scores. Rollard’s review also found that high achievement and student effort come when teachers and teaching actively promote these things. Sullivan et al. (2015) link this to “teachers regularly posing challenging tasks, as well as explaining the purpose of the challenges, and encouraging students to persist, emphasising the connections between persistence and learning and affirming persistence when they see it” and they have investigated the types of lesson structures and supports for teachers and students that work. Resources demonstrating such features are required.

When teachers use real world problems that capitalise on issues of local and timely interest for students, students see that mathematics can be for them. It is important that teachers are helped to acquire the skills to create resources or modify existing resources to connect with their own students, wherever in Australia they live.

Because many students do not realise their potential in mathematics, they need opportunities to catch up to their peer group, and to build self-esteem. Sample approaches are the Yumi Deadly programs for accelerating Indigenous students and low SES students from a low base to age-level norms, and the Quicksmart program for improving fluency in number skills (See Question 5). The AAMT: *Make It Count: Maths and Indigenous Learners* (<http://mic.aamt.edu.au/>) provides resources to support schools in developing their own innovations for improving mathematics learning outcomes of Aboriginal and Torres Strait Islander students. The *Maths in the Kimberley* project (Griffith University) reported that over three years there had been improvements in certain pedagogical aspects of lessons, such as higher order thinking and assessment for learning, but no change in others, such as multiple entry points, group work, and connections within mathematics and to other curriculum subjects (Jorgensen, Grootenboer, Sullivan & Nietzsche 2010). Evaluation of the New South Wales Board of Studies *Maths in Indigenous Contexts* project (<http://ab-ed.boardofstudies.nsw.edu.au/go/resources/numeracy-development>) has shown

that the project was successful for some students and some schools, but there needs to be strong and active leadership at the local school community level if the project is to be successful.

Many digital resources have a strong visual element, providing support for students with learning difficulties and NESB students. There are many applets and games for mathematics for computers and tablets. Many of these will be very attractive to low achieving students, so checking pedagogical quality is essential. Engagement has to be cognitive, not just affective.

Recommendations. Attending to these issues is a general feature of all projects within the co-ordinated overall approach, especially projects 1, 2, 4, 5 and 7.

Question 8: Where might specific targeting of resources be most helpful? (Consider age groups, numeracy skill acquisition)

The answers to Questions 6 and 7 justify:

- Addressing difficulties that teachers experience in assembling coherent programs that address the full goals of the ACM from the many resources available. (Project 6)
- Addressing the major gap related to problem solving and reasoning, including classroom culture and challenge, and avoiding the shallow teaching syndrome. Helping teachers use challenging problems, attending to productive disposition (e.g. persistence), and making mathematics relevant to students' current interests and future lives. (Projects 1 and 2)
- Attending to real world problem solving and mathematical modelling, including a special focus on formulating problems mathematically and exploiting local situations. (Project 1).
- Secondary geometry digital resources including for reasoning and proof (Projects 5 and 8)
- Updating excellent early numeracy programs and making available around Australia. (F-3) (Project 9)
- Intervention for building confidence, self-concept and strengthening procedural fluency in students whose low achievement has been due to social factors (Low SES, Indigenous) (Projects 4 and 7)
- Broadening the bandwidth of recognised summative assessment, promoting in-class formative assessment and providing on-line diagnostic assessment of concepts (Useful for all) (Project 2)

Question 9: How well do existing resources support out-of-field teachers in developing their mathematics teaching skills?

Existing resources play a very small role in supporting out-of-field teachers to develop their mathematics teaching skills, mainly by assisting teachers to brush up on the content which they have to teach. This is a very important problem because the most recent TIMSS survey (Thomson, Hillman & Wernert 2012) showed that around a third of Year 8 students are taught by out-of-field teachers and teachers of around one fifth of the students said that they were no more than ‘somewhat confident’ in teaching mathematics. Out-of-field teaching (by definition only in secondary schools) is most prevalent for the students who need good teaching the most—in low SES and regional or remote areas. In these areas, it is not uncommon for *all* the mathematics teachers to be teaching out-of-field. The staffing in some of the schools is also very unstable. Out-of-field teachers are likely to ‘text-book teach’, place emphasis on skill development, lack knowledge to find and use other resources, lack the MPCK to distinguish between good and poor resources, and many simply cannot undertake challenging questions in the classroom.

Out-of-field teachers are difficult to reach with professional learning, especially those who have only a small and changing part of their teaching load in mathematics. Thus they may not have much time to put into their mathematics teaching and do not see it as the key part of their professional identity. The real solution is to fix supply, but this is unlikely to be achieved.

Some ‘out-of-field’ teachers have a reasonably strong MCK but no mathematics specific pedagogical training. Some have insufficient mathematics to confidently tackle the senior mathematics where the school needs a teacher. Others have very limited MPCK and limited mathematics background and do not have good understanding, even of the mathematics that they are teaching. For example, the authors of this report taught in two post graduate certificate courses for out-of-field teachers at the University of Melbourne; one course for Years 7–10 teachers and one for Years 11 and 12 teachers. None of us had predicted the extent of difficulty that some of the participants had with fundamental number and algebra concepts from Year 8 up, and the absence of geometry knowledge. These courses, based on MCPK and MCK, were highly successful for those who completed, but completing such a course whilst working full time was extremely difficult for some. Despite all primary teacher education courses in Australia including mathematics education, some primary teachers share the characteristics of out-of-field secondary teachers: insufficient MCK and MPCK to do their work.

The general thrust in teacher professional development in recent years has been to support school-based groups working together (e.g. through mathematics coaches, lesson study etc.) so that the impact is directly on the work of teaching at that school and team improvement of the whole school program. This is the preferred model where it is feasible. However, many out-of-field teachers are not working with mathematically strong colleagues and located away from major cities, so on-line courses for them have significant advantages in both time and money. It is our judgement that online professional development has a role, but the more mathematical aspects of both MCK and MPCK require some face-to-face support. Even basic technical aspects, such as the difficulty of real-time input of mathematical symbols and diagrams, make face-to-face mathematical discussion easier. Hence a blended model is recommended. Research (U.S. Department of Education, 2010) on on-line professional development shows that a blended approach is

generally better than either entirely face-to-face or entirely online, provided the design capitalises on the different strengths of the online and face-to-face components. Reports such as Allen et al. (2011) recommend features for effective on-line course design.

We recommend the following resource development:

- Creating a resource to improve mathematical pedagogical content knowledge of teachers (including consolidating and refreshing existing materials) supported with blended learning. (Project 3).
- Well supported award courses for out-of-field teachers that focus on MPCCK and a deep understanding of the content which they will teach with some teaching release for participants. Separate courses for Years 7 – 10 and senior mathematics. Blended learning with face-to-face and online components. (Project 11).
- Similarly focussed primary courses could be offered, although this need really should have been addressed in initial teacher education.
- Later consideration of providing a fully resourced program with accompanying professional development for Years 7 – 10 for use in schools with staffing difficulties. (Not a current recommendation).

Question 10: How well do existing resources focus on cognitive skills in addition to mathematical content knowledge and skills—enabling students to deal with complex situations, explore, make and test conjectures, reason logically, and use a variety of mathematical methods to solve problems?

This has been the theme of much of this report. Whilst there are many excellent resources for the higher-order thinking that is described in the question, we know that teachers have difficulty in using them well, hence the recommended emphasis above on a pedagogy that assists this, and well-engineered and structured lessons that work. Stein, Grover & Henningsen (1996) describe how lessons focussed on higher-order thinking are commonly derailed, and the challenge is diminished. Some of the poor results on PISA are likely to be directly due to students not having been taught, or experiencing, mathematical modelling and formulating problems mathematically.

Recommendation: Projects 1, 2, 3, 5 and 8 address this.

Question 11: To what extent do resources and pedagogies provide teachers with the ability to teach mathematical skills and understandings in ways that encourage transfer—embedding real-world work-related examples and technologies into lessons?

This is also strongly related to the theme of this report. The structure of mathematics curricula set out in content strands, and the design of mathematics resource books in highly self-contained chapters contributes to the perception by students and teachers without a strong mathematical background that mathematics consists of isolated rules and procedures. The tick-the-box approach to delivering the ACM in many schools also contributes. Teachers need help making connections between mathematical concepts and seeing the big ideas. Students need more problems where they are expected to bring together diverse mathematical knowledge. The Victorian *Challenging Problem* and *Problem Solving and Modelling* tasks used in the Victorian Certificate of Education Mathematics subjects in the 1990s showed how transfer could be achieved (see Question 3).

A related issue is that Australian teachers and textbooks have only weak strategies for helping students solve problems expressed in words, in other words for formulating mathematical models. Addressing this important gap (related to PISA performance) is a focus of Project 1.

Recommendation. Project 1 is especially designed for transfer, and it is a strong element of Projects 2 and 3.

Question 12: Is there a need for fresh approaches/resources to be developed? If so, why? If so, what should such resources cover—what do out-of-field teachers most need?

The major fresh approach relates to the seamless integration of digital technologies with mathematical capability. Digital technologies present excellent opportunities for improving mathematics teaching and learning in multiple ways. In most schools, regular use in class and at home is now becoming practical. Computers and hand held devices can provide surface attractiveness (not to be underestimated) such as colour; serious tools for doing and visualising mathematics and enabling students to do more by taking away the burden of computation; games to practise number skills and to develop strategic thinking; and portraying contexts for problem solving. There are real world interface technologies (e.g. to analyse photos (shapes) and videos (motion), get data from temperature probes) to bring the real world into the classroom; dynamic geometry software such as GeoGebra to simulate motion and explore the geometry in everyday objects (such as car jacks, scissor lifts, ironing boards), videos to display authentic problems for students to engage in mathematical modelling, and access to large data sets and the tools to interrogate them. There are tools to support students in research, communication and collaboration.

Using digital technology in these ways should now be included in all resources.

Recommendations. Resources specifically targeting digital technology would be included in Project 1, 5 and 8. Use of digital resources would be a widespread feature of projects within the overarching initiative.

To make sure mathematics does not stay in 20th century mathematics, we also recommend:

Investigation by ACARA of a new Year 12 subject using mathematically-able software to amplify what students do, and including relatively new mathematical topics that have arisen in the digital era, such as the analysis of 'big data', visualisation, animation, biology etc.

Approaches not recommended

There are often calls for completely fresh approaches to teaching mathematics, but in our opinion they are not good investments. One suggestion is that inquiry-based learning be used in its more student-centred versions, where students have a lot of control over the problems that they want to study, and the solution methods and hence what they learn and the teacher is a 'guide on the side'. This form of inquiry learning has a place in some special settings, but there is no research evidence to suggest this is a good direction for curriculum reform. Instead research supports an active role for teachers selecting well engineered problems, and encouraging and supporting solutions that advance towards desired mathematical methods.

Another suggestion springs from observation that some young people become totally absorbed in computer games where the player is immersed in a virtual world, often for very long periods of time. Educators such as Gee (2005) have analysed the characteristics of these digital environments and called for significant cross-over into schools. We have searched the internet and asked our contacts for examples of games such as this from which it is likely that students will learn significant mathematical content, beyond practising number skills. However, we have not found one such game. We therefore think this would not be a high priority investment at this stage. A step down from these immersive virtual worlds, are games with specific mathematics content (perhaps disguised), which support teaching of a lesson or unit of work, such as promoted by Devlin (2011) and discussed above. Many of these are excellent and should be used more (see Questions 1 and 6).

GAP ANALYSIS OF EXISTING EDUCATIONAL RESOURCES

Question 13: Where are the gaps in existing teacher and student resources available online to support the Australian Curriculum: Mathematics?

As noted above, the quality of online resources is too variable, so greater screening for quality is needed. Hidden in there are some real gems (e.g. wonderful games encouraging the development of concepts, skills and strategic thinking). Most online resources are of ‘one-off’ activities, and links to the ACM only partially reveal sensible sequencing. There is a need for attention to creating coherent programs which make good use of digital resources within a curriculum that promotes all proficiencies of the ACM. Some other specific gaps have been identified above.

Consider whether there are sufficient free, high-quality mathematics resources for primary school teachers

Many teachers (both primary and secondary) need assistance with the conceptual understanding proficiency and seeing the many topics of the ACM as a connected whole. A new resource is needed and existing ones consolidated to improve teachers’ MPCK of mathematical concepts, based on the best research evidence. This will include knowledge of students’ stages of learning of major topics, likely difficulties and misconceptions, the internal connections within the topic and its place within the big ideas that run through the curriculum and the major resources for teaching and how to use them (e.g. place value materials, dual number line, explorations with graphics calculators). A strong basis for this already exists on government websites (e.g. Victoria’s *Scaffolding Numeracy in the Middle Years* on multiplicative thinking and the *Mathematics Developmental Continuum*) and resources from teacher education (e.g. Teaching Number in the Middle Years extranet.education.unimelb.edu.au/SME/TNMY/ and the smart tests (Stacey et al., n.d.).

Consider whether there are sufficient engaging mathematics resources for junior secondary students

As before, this is probably a perception rather than reality. Consideration could be given to creating a complete program for Years 7 – 10 which incorporates all proficiencies and makes exemplary use of digital technology in the ways described above. This would help out-of-field teachers and schools with unstable staffing. See also Question 6 and elsewhere. Remember cognitive engagement is the highest priority, although affective and behavioural engagement link to it.

Consider whether existing curriculum resources utilise emerging digital technologies to illustrate mathematical concepts e.g. graphical or numerical programmes that can assimilate large amounts of data to illustrate changes to inputs or outputs

There is a strong research base that demonstrates how such mathematically-able software (as described above) can be used to best advantage. Most resources and textbooks now tend to use mathematically-able software (on computers or calculators) only for calculation. However there are many pedagogical opportunities (Pierce & Stacey, 2010).

As well as illustrating mathematical concepts, it can support problem solving and reasoning, change the classroom dynamics and the didactic contract and support student exploration and discovery. It enables analysis of real large scale data, such as the data from students which was provided by the Australian Bureau of Statistics *CensusAtSchool* website: (www.abs.gov.au/censusatschool). (We understand that this valuable resource has lost ABS funding, but another source of funding is very likely.) Using such tools to pedagogical advantage is essential in all new resources.

The gaps in resources to develop reasoning in Years 7 – 10A, particularly for geometry, could be partially filled by a new resource of carefully designed dynamic geometry applets. An example of the quality of resources needed is from Ireland's RCEMTL Junior Certificate Geometry Applets (<http://www3.ul.ie/cemtl/resources.htm>). This is a well organised, easily navigable website representing the type of resource that is needed for the Australian Curriculum. The geometry applets present proofs but they could be modified to provide scaffolding for students to develop their own proofs. The NSW Geometer resource available via Scootle also provides many dynamic geometry files but these show only properties, not reasons.

Recommendations. Projects 5 and 8 specifically address these needs. It is recommended that a first stage for Project 5 is to audit the RCEMTL and other sites (e.g. Geogebra Tube) with a view to reducing applet development costs.

Are there other gaps in resources for teachers and/or students?

Many education systems now have libraries of lesson videos focusing on particular topics that teachers know are difficult to teach or difficult aspects of pedagogy. The Annenberg Learner resource of 60 videos *Teaching math: A video library K–4 and 5–8* are a good example. Each lesson video is accompanied by teacher professional development resources relating to the lesson task, including links with the curriculum content and process standards. This particular resource is designed for the USA and is not free (www.learner.org/resources/series32.html).

Recommendation. Because of the likely expense, a general video project is not recommended. However, some projects (e.g. Projects 2 and 3) will use videos in associate professional learning.

Consider whether the gaps could be filled by existing resources that are not formally linked to the Australian Curriculum (in use in Australia or overseas) (including consideration of whether such resources are free for education, evidence of teacher satisfaction and evidence of improved student learning outcomes)

There are many examples above of promising resources. Please note however that tight evidence for quality, teacher satisfaction and improved student learning outcomes is only rarely available. Moreover, research evidence and thorough evaluation has not been able to be sought within the time scale for this project. We believe that many of the resources cited above are excellent and should be used in Australia (especially the digital learning objects many of which are already used in some schools). Others need modification for Australian conditions (e.g. ACM terminology) whilst the others have potential to act as models for Australian resources. For example, the *Mathematics Assessment Project* requires modification for ACM and its assessment processes, review of all resources for appropriateness with subsequent replacement or modification, and an overarching concept that provides a very strong model for Australian work (e.g. in Project 2).

Recommendation. Many existing resources are directly useful in the recommended projects, or provide useful models.

Consider which age groups could most benefit from the development of new types of resources and the rationale/evidence for such approaches

Covered above.

Question 14: Are there any other issues of critical importance?

1. Quality resource development must be supported with adequate budgets and realistic timeframes to permit clear conceptualisation, precise needs analysis, creative materials design, insightful field testing, expert review and editing in order to build Australian capacity to produce mathematics materials of international acclaim. Too many resources, including some on respected websites, do not demonstrate high standards. Of course, there is no evidence that students learn maths better from materials without spelling, grammatical or small mathematical mistakes, but minor infelicities create an impression that excellence does not matter. Issues with other resources go much deeper.
2. We have reservations about emphasis on free resources: it encourages teachers to engage in unsystematic dipping with little evaluation; and good on-line resources need technical maintenance and updating. Students need a coherent program, and should keep a written record (maybe on a digital device) of what they learn. This is difficult with digital learning objects from many sources.
3. A body of research is accumulating on how to achieve large scale change for mathematics. For example, the *Middle-school Mathematics and the Institutional Setting of Teaching* project (Cobb, not dated) has studied large scale mathematics change processes over many years with four US school districts. They found that widespread improvement

can only be achieved if all parts of the system (districts, school leaders, mathematics specialists and teachers in professional development networks) work together on a coherent instructional system comprising

- explicit goals for students' mathematical learning, a detailed vision of high-quality instruction, and curriculum compatible with this vision
- professional development (from the district mathematics specialists and extended within the school) that is organized around the curriculum materials being used within the schools
- assessments aligned with the goals for students' mathematical learning
- additional support for struggling students to enable them to succeed in mainstream mathematics classes.

We feel that this multi-faceted approach, based around a coherent set of curriculum materials is missing. In mathematics in Australia, it has come closest to being achieved within the early years' number programs about 15 years ago. Ms Cathy Beesey of the Victorian DET reports that internal evaluations show that the introduction of mathematics specialists to primary schools has been powerful in building capacity and improving learning beyond expected levels.

Recommendation. Conduct a review of research on achieving large-scale change in mathematics. (Project 10).

RECOMMENDED ACTION

In responding to the questions above, many needs have been identified. We suggest a co-ordinated approach, with a long term agenda, and implementing prioritised actions. Within this, we suggest three major projects and 8 smaller projects, in priority order. Three recommendations for the ACM are listed separately.

Over-arching recommendation: Co-ordinated research and development

Institute a long-term Australian initiative of *co-ordinated* materials development and evaluation, with associated research and professional learning for system-wide improvement in mathematics education.

Projects within the initiative will be held together by some important common features. They will promote a common approach to mathematics where inquiry, problem solving and reasoning feature strongly, with students actively engaged in thinking mathematically and exercising 21st century skills of communication, creativity, collaboration and critical thinking. Across all the projects, materials are to be ‘transformative’, using high quality student resources as a capacity-building vehicle for teacher learning. Quality is assured through careful research and development, detailed trialling in appropriate schools, expert review and incorporating established research findings. A consistent pedagogical approach is promoted, which attends to challenge, engagement and equity. Imaginative tasks will frequently use new technologies for multiple purposes including computation, teaching concepts and to bring the real world into the classroom. Ideally most of these projects would be accompanied by blended on-line and face-to-face professional learning.

Research projects within the initiative are to provide practical advice for educational authorities and schools on decisions about mathematics programs and within projects to guide resource development.

Major projects in priority order

Project 1: Solving real world problems mathematically

Create, refresh and consolidate resources to teach students to solve problems set in real world contexts (in the spirit of PISA’s mathematical literacy), especially focusing on formulating mathematical models. This project fills a gap in Australian teaching methods, and responds to falling scores on the PISA survey. It requires developing and/or adapting from other countries specific teaching methods (e.g. Asian focus on problem formulation) and techniques for students to identify variables, relationships and assumptions to build mathematical models (e.g. Singapore model method in primary years; others later). This project could use digital technologies to bring the real world into the classroom (videos, probes, simulations, data) and mathematically able software. It could also provide advice for teachers on capitalising on local events and timely issues in the way they teach mathematics, which is relevant to students who feel alienated from mathematics. The project would provide a small number of new well-researched teaching methods, for teachers to use across multiple ACM strands and year levels. Hence it is feasible to work from Years 3 to 10.

Project 2: Assessment to drive a more complete interpretation of the ACM

Develop well-supported resources for teacher/school assessment of the full bandwidth of mathematical proficiencies (especially the neglected problem solving and reasoning

proficiencies) to supplement NAPLAN. Consider means of providing such an assessment with external status, so that it sits alongside NAPLAN as an important indicator of school mathematics achievement. The intention is to use assessment to drive attention to teaching problem solving and reasoning ACM proficiencies in schools. The project includes creating, refreshing and consolidating exemplary tasks, lessons, and associated professional development to address pedagogy. Assessment methods can be chosen to promote productive disposition and student engagement. The Mathematics Assessment Project is a model of such a comprehensive approach, and some of its materials may be modified for use. Priority Year levels: (1) Years 7 and 8; (2) Years 9 and 10; (3) Years 5 and 6.

Project 3. Supporting expertise in mathematics teaching

Consolidate, refresh and create resources to improve mathematical pedagogical content knowledge of teachers, especially for teaching mathematical concepts, based on best research evidence. The project can draw on some of the principles of the large early numeracy projects (e.g. clear directions from explicit stages of learning, diagnostic and formative assessment, well selected student resources), incorporate existing materials from several different sources, and reveal the ‘big ideas’ running through the ACM. Materials will include readable research summaries, diagnostic assessment, sample lessons, appropriate pedagogy, etc. Priority year levels: (1) Years 7, 8, 9 and especially out-of-field teachers; and (2) Years 5 and 6.

Smaller projects in priority order

Project 4. Conduct a study to identify the most educationally effective, cost-effective and sustainable ways that schools in low SES communities are supporting students with significant learning gaps (frequently Indigenous and disadvantaged students) to catch up. Attend to procedural fluency, conceptual understanding, confidence, self-efficacy, engagement and subsequent success, Consider support in and out of the classroom. Prepare advice for other schools.

Project 5. (Fill a gap.) Create a new resource and associated on-line teacher professional development for teaching reasoning through geometry, using carefully designed dynamic geometry applets. Applets will demonstrate properties, and present dynamic proofs, and support students to develop their own proofs. Some applets can be sourced from current sites. Priority: Years 9 – 10A.

Project 6. Provide models, advice and teacher development for creating whole school programs to deliver the ACM, with careful attention to sequencing and connections between topics, assessment and incorporating all the proficiency strands. Investigate the information that teachers need about resources (especially digital learning objects) in order to make sound selections in a time efficient manner for use by Scootle and similar services.

Project 7. Conduct a review of the various projects working with regional/remote Indigenous students to determine educational effectiveness, cost-effectiveness and sustainability, in order to provide advice to education authorities and schools. Priority: All year levels.

Project 8. (Fill a gap.) Create, refresh and consolidate resources to build secondary teachers’ capacity to use a basic range of mathematically-able software (calculator and computers; spreadsheets, dynamic geometry, computer algebra) both to amplify what

students can do and to exploit the pedagogical opportunities it provides to promote better learning. Priority 1: Years 10 – 12. Priority 2: Years 7 – 9.

Project 9. Consolidate, update where required and make available to all Australian teachers, the resources of the early year numeracy projects. Priority Year Levels: F-3

Project 10. Conduct a review of the growing body of research evidence on methods for achieving large scale change for mathematics, including their educational impact, sustainability, costs etc.

Project 11. HIGHER EDUCATION Provide two well-supported, strictly assessed award courses for out-of-field teachers that focus on MPCK and a deep understanding of the content which they will teach. Deliver courses through on-line learning and school holiday programs, and provide some teaching release for participants for study. One course should be on Years 7 – 10 and the other on Years 11 and 12. A similar course for upper primary teachers could be considered. Conduct formative and summative evaluations of the courses and subsequent impact in schools.

Recommendations related to Australian Curriculum Mathematics

- Regularly and systematically review, improve and update the ACM, with due consideration to the required balance between stability and improvement in the changing digital environment.
- Better describe problem solving and reasoning proficiencies in the ACM to explain what students have to learn about problem solving and reasoning, beyond the content being used.
- Appoint a task force to investigate new priorities for the senior mathematics curriculum arising from new areas of application of mathematics (e.g. to biology, security, large data sets, data visualisation, automation) and using mathematically-able software to amplify what students can do.

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